

# Linear Algebra Self-Assessment

CS 135: Computational Semantics · Fall 2026

Handed out Session 1 · Self-scored · Report section scores by Monday night (8/31/26)

---

**This is not graded, and it is not a test of whether you belong in this course.** It is a map. The second half of CS 135 uses a specific and fairly narrow slice of linear algebra, and this assessment tells you — and us — which parts of that slice you already know and which you should spend a few hours on before Week 7. Students arrive in this course from computer science, linguistics, and elsewhere, with very different backgrounds. That is expected and completely fine.

**How to take it.** Work without notes, references, or AI assistance — a wrong answer here costs you nothing and tells you something useful, while a looked-up answer tells you nothing at all. Expect 30–40 minutes. If a question means nothing to you, write “don't know” and move on; that is a perfectly good answer and more useful to us than a guess.

**Before you check each section against the key, circle your confidence for that section.** The gap between confidence and score is often more informative than either one alone.

**Afterwards.** The answer key and review guide will be posted Friday evening. Score yourself, then submit your six section scores and confidence ratings using the form linked on the course site by **Monday night**. Individual results are not graded and do not affect your grade in any way; we use them to calibrate the pace of the geometric algebra unit and to point people at the right review material.

## Section A — Vectors and Vector Spaces

*Used throughout; Session 12 in particular.*

**A1.** Let  $\mathbf{u} = (2, -1, 3)$  and  $\mathbf{v} = (1, 4, -2)$  in  $\mathbb{R}^3$ . Compute  $3\mathbf{u} - 2\mathbf{v}$ .

---

**A2.** Which of the following is a vector space under the usual operations?

- (a) All  $(x, y, z)$  in  $\mathbb{R}^3$  with  $x + y + z = 0$
  - (b) All  $(x, y, z)$  in  $\mathbb{R}^3$  with  $x + y + z = 1$
  - (c) All  $(x, y, z)$  in  $\mathbb{R}^3$  with  $x \geq 0$
  - (d) More than one of the above
- 

**A3.** True or false, and explain in one sentence: *a vector in  $\mathbb{R}^3$  is a list of three numbers.*

---

Before checking answers — confidence in this section (circle one): 1 2 3 4 5

## Section B — Span, Independence, Basis

Used in Sessions 13 and 17 — blades represent subspaces.

**B1.** Are  $\mathbf{u} = (1, 0, 1)$ ,  $\mathbf{v} = (0, 1, 1)$ ,  $\mathbf{w} = (1, 1, 2)$  linearly independent in  $\mathbb{R}^3$ ? Show why or why not.

---

**B2.** What is the dimension of  $\text{span}\{(1, 2), (2, 4), (3, 6)\}$  in  $\mathbb{R}^2$ ?

---

**B3.** How many vectors are in a basis for  $\mathbb{R}^5$ ?

---

**B4.** Suppose  $S = \{\mathbf{v}_1, \dots, \mathbf{v}_7\}$  spans  $\mathbb{R}^4$ . What can you conclude about  $S$ ?

---

Before checking answers — confidence in this section (circle one): 1 2 3 4 5

## Section C — Inner Products, Norms, Orthogonality, Projection

Used in Sessions 7–9 (embedding similarity) and Session 14 (contraction and projection).

**C1.** Compute  $\|\mathbf{u}\|$  for  $\mathbf{u} = (3, 4)$ .

---

**C2.** Compute  $\mathbf{u} \cdot \mathbf{v}$  for  $\mathbf{u} = (1, 2, 2)$  and  $\mathbf{v} = (2, -1, 0)$ . What does your answer tell you about the two vectors?

---

**C3.** For nonzero  $\mathbf{u}$  and  $\mathbf{v}$ , what does  $\mathbf{u} \cdot \mathbf{v} = 0$  mean geometrically?

---

**C4.** Compute the projection of  $\mathbf{u} = (3, 4)$  onto  $\mathbf{v} = (1, 0)$ .

---

**C5.** Compute the cosine similarity of  $\mathbf{u} = (1, 1)$  and  $\mathbf{v} = (1, 0)$ . What angle does this correspond to?

---

Before checking answers — confidence in this section (circle one): 1 2 3 4 5

## Section D — Matrices as Linear Maps

Used in Sessions 8–9 (embeddings, attention) and Session 11 (tensor products).

**D1.** Let  $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$  and  $\mathbf{x} = (1, 1)$ . Compute  $A\mathbf{x}$ .

---

**D2.** If  $A$  is the matrix of a linear map  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ , what do the *columns* of  $A$  represent?

---

**D3.**  $A$  is  $3 \times 2$  and  $B$  is  $2 \times 4$ . What are the dimensions of  $AB$ ?

---

**D4.** True or false: for square matrices  $A$  and  $B$ ,  $AB = BA$  in general.

---

Before checking answers — confidence in this section (circle one): 1 2 3 4 5

## Section E — Determinants, Area, and Orientation

The most important section for this course. Sessions 12–13 build directly on it.

**E1.** Compute  $\det \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ .

---

**E2.** Geometrically, what does the *absolute value* of a  $2 \times 2$  determinant measure?

---

**E3.** Suppose  $\det[\mathbf{a} \ \mathbf{b}] = 5$ , where  $\mathbf{a}$  and  $\mathbf{b}$  are the columns. What is  $\det[\mathbf{b} \ \mathbf{a}]$  — the same matrix with its columns swapped? What does the change represent?

---

**E4.** If  $\det A = 0$  for a  $2 \times 2$  matrix  $A$ , what does that tell you about its columns?

---

**E5.** What does the absolute value of a  $3 \times 3$  determinant measure?

---

Before checking answers — confidence in this section (circle one): 1 2 3 4 5

## Section F — Rotations and Orthogonal Transformations

Used in Session 15 — rotors are the geometric algebra treatment of rotation.

**F1.** Write down the  $2 \times 2$  matrix that rotates the plane counterclockwise by angle  $\theta$ .

---

**F2.** What property characterizes a linear transformation that preserves all lengths and angles? Give the matrix condition if you know it.

---

**F3.** In three dimensions, do rotations about different axes commute — that is, does the order in which you apply them matter?

---

Before checking answers — confidence in this section (circle one): 1 2 3 4 5

### **Section G — Optional (not prerequisite)**

*These are taught in the course, not assumed. Answer only if you already know them — it helps us pitch Part II.*

**G1.** In one sentence: what does singular value decomposition (SVD) do?

---

**G2.** In one sentence: what is an eigenvector of a matrix?

---

### **When you are done**

Check your work against the key posted on the course site Friday evening, score each section, and submit your six section scores and confidence ratings via the linked form by Monday night. The key includes a review guide pointing you at specific material for whichever sections came out weakest.

If several sections came out low, that is worth a conversation rather than a worry — please come to office hours in the first two weeks. There is plenty of time to close a gap before Week 7, and considerably less time once we are in the middle of the geometric algebra unit.