

Sample Problems for Quiz 1

CS 135: Computational Semantics – From Logic to Geometry

Brandeis University · Fall 2026

What this is. These are not the quiz questions. They are the same *kinds* of task on different data, drawn from the same material – so working them is practice rather than memorization. The quiz has four sections in this order, and so does this sheet.

How to use it. Work each problem with a pen before looking at the answer. Every number here is small enough to do by hand, and you may leave an answer as $\log_2$ of a fraction, or as an unsimplified fraction, wherever that is the exact form of it. **No calculator is needed on the quiz and none is permitted**, so do not use one here either.

If you can do every problem on this sheet, you are prepared. If one of them stops you, that is the thing to ask about before the quiz.

Given. These are printed on the quiz as well. You are not being tested on whether you memorized them.

$$\text{PMI}(w, c) = \log_2 \frac{p(w, c)}{p(w)p(c)}$$

$$\text{PPMI}(w, c) = \max(0, \text{PMI}(w, c))$$

$$\cos(\mathbf{u}, \mathbf{v}) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

$$p_\alpha(c) \propto \text{count}(c)^\alpha$$

Useful for hand computation: the association ratio can be written entirely in counts, with no probabilities, as $\frac{p(w, c)}{p(w)p(c)} = \frac{N \cdot n(w, c)}{n(w)n(c)}$.

1. Sets, relations, and types

Throughout this section the domain is fixed:

$$\mathcal{D}_e = \{ \text{Mia}, \text{Noah}, \text{Omar}, \text{Jaco} \}$$

with coordinates in that order, so $\mathbf{e}_1 = \text{Mia}$ and $\mathbf{e}_4 = \text{Jaco}$ (a dog). Two predicates:

$$\text{teacher} = (1, 0, 1, 0), \quad \text{travels} = (1, 1, 1, 0)$$

and one relation:

Q (<i>trusts</i>)	Mia	Noah	Omar	Jaco
Mia	0	1	0	1
Noah	1	0	0	0
Omar	0	1	0	1
Jaco	0	0	0	0

$Q_{ij} = 1$ when individual i trusts individual j . Rows are the one who trusts.

A1. Compute $\text{teacher} \odot \text{travels}$ and say in English which set it denotes. Then compute $\text{teacher} \cdot \text{travels}$ and say what the number counts.

A2. Evaluate *Every teacher travels* and *Every traveler teaches*, using only cardinalities. State the test you applied, and explain why the two sentences do not get the same answer.

A3. Evaluate *Some teacher travels* and *No teacher travels*. Give the test in each case.

A4. Compute $Q \mathbf{e}_{\text{Noah}}$ and $Q^\top \mathbf{e}_{\text{Noah}}$. Name each set in English, and say in one sentence why they are not the same set.

A5. Is *Omar trusts Jaco* true? Write the sentence as a bilinear form before you evaluate it. Then do the same for *Jaco trusts Omar*.

A6. Evaluate *Every teacher trusts Jaco*. You will need to build one of the two sets first.

Hint. A quantifier needs two sets of individuals. The scope here is not given to you directly – extract it from Q .

A7. Give the semantic type of *trusts* and the tensor order of the object representing it. What order results from applying it to one entity, and what type is that result?

A8. A sentence of the form *Noah trusts Mia* evaluates to a single number. In terms of tensor order, account for how two order-1 objects and one order-2 object produce an order-0 object.

2. The distributional pipeline

A term–context co-occurrence table over three words and three contexts. Margins and total are given, so you never have to add a row up yourself.

	c_1	c_2	c_3	row
w_1	150	25	25	200
w_2	25	50	25	100
w_3	25	25	50	100
col	200	100	100	$N = 400$

B1. Compute $\text{PMI}(w_1, c_1)$ and $\text{PMI}(w_2, c_1)$. Leave each as $\log_2$ of a fraction, and give a decimal only where it is exact.

B2. Compute the whole PMI row for w_2 . One of the three entries is exactly zero – say what that zero means about w_2 and that context.

B3. Write out the PPMI matrix for the whole table. Then describe its shape in one sentence, and say what PPMI has destroyed in this particular table.

B4. Compute $\cos(w_2, w_3)$ on the PPMI rows. Then compute it on the *raw count* rows. The second one is an exact fraction; find it.

Hint. For the raw rows, notice that w_2 and w_3 have the same norm.

B5. You have just computed $5/6$ and 0 for the same pair of words. Explain why the two numbers disagree so sharply, in terms of what each one is measuring.

B6. Under PPMI, all three pairwise cosines in this table come out to 0 . Is that a bug in the data, a bug in PPMI, or neither? What does it suggest about the next step in the pipeline?

B7. The context counts are $(200, 100, 100)$. Applying context distribution smoothing with $\alpha = 0.75$ gives $p_\alpha \approx (0.457, 0.272, 0.272)$, where the unsmoothed p was $(0.50, 0.25, 0.25)$. The PMI row for w_2 changes from $(-1, +1, 0)$ to approximately $(-0.87, +0.88, -0.12)$. Account for both the direction of the change and the reason anyone would want it.

3. Metrics and bilinear maps

C1. Let $\mathbf{u} = (2, 3)$ and $\mathbf{v} = (4, 1)$.

(a) Compute $\mathbf{u}^\top I \mathbf{v}$.

(b) Compute $\mathbf{u}^\top G \mathbf{v}$ for $G = \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$, writing out all four terms of the sum before you add them.

C2. A query $\mathbf{q} = (3, 1)$ is compared against two candidates, $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 4)$.

(a) Under the ordinary dot product, which is returned?

(b) Find the range of g for which $G = \text{diag}(1, g)$ reverses that ranking, and verify one value of g by computing both scores.

(c) What happens at the boundary value of g ?

C3. Let $\mathbf{u} = (2, 1, 0)$ and $\mathbf{v} = (1, 0, 3)$. Write out $\mathbf{u}\mathbf{v}^\top$ as a 3×3 matrix, state its rank, and say why the rank had to come out that way.

C4. Still with $\mathbf{u} = (2, 1, 0)$ and $\mathbf{v} = (1, 0, 3)$: compute $\mathbf{v}\mathbf{u}^\top$, and state its relationship to $\mathbf{u}\mathbf{v}^\top$. Why does this matter for representing a relation?

C5. Take $\mathbf{u} = (1, 0)$ and $\mathbf{v} = (0, 1)$.

(a) For $G = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, compute $\mathbf{u}^\top G \mathbf{v}$ and $\mathbf{v}^\top G \mathbf{u}$.

(b) Do the same for $G' = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(c) State the general fact your two answers illustrate.

C6. Let $\mathbf{u} = (1, 2)$ and $G = \text{diag}(4, 1)$. Compute $\|\mathbf{u}\|^2$ and $\|\mathbf{u}\|_G^2 = \mathbf{u}^\top G \mathbf{u}$, and say in one sentence what a change of G does besides reranking neighbors.

C7. Return to the matrix Q (*trusts*) from Section 1. Is Q symmetric? Name a specific pair of individuals that settles the question, and then name a different pair for which Q happens to be symmetric. What does the asymmetry represent linguistically?

4. Essay practice

On the quiz there is one short essay, chosen from two prompts, worth 10 points and expected to run about a bluebook page. It is marked on whether the argument is correct and specific, not on length.

The three prompts below are on the two themes the quiz essay draws from. Try at least one of them in writing, under time.

D1. A classmate says: “cosine similarity measures how similar two words are in meaning.” Give the most precise statement you can of what cosine actually measures in the distributional regime, and name two things it does not measure.

D2. In Session 6 the inner product $\mathbf{u} \cdot \mathbf{v}$ computed a cardinality, and from it a truth value. In Session 7 the same arithmetic computed a similarity score. Nothing about the formula changed. Account for the difference, and say what was given up in the exchange.

D3. You are told only that a proposed composition operation takes two word vectors to a phrase

vector and is commutative. Without knowing anything else about it, name a linguistic phenomenon it is guaranteed to get wrong, and explain why this is a matter of algebra rather than a failure of training.

A checklist, if you prefer one. By the day of the quiz you should be able to, without notes:

- turn a set into a vector and back, and say what $\odot$ and $\cdot$ each compute;
- evaluate EVERY, SOME and NO from two cardinalities, and say which of them care about the restrictor;
- get a row or a column out of a relation matrix, and say in English which set you have;
- write a transitive sentence as a bilinear form;
- give the type and tensor order of a predicate, a relation, and a sentence, and say what application does to the order;
- compute a PMI cell from counts alone;
- say what clipping at zero fixes and what it costs;
- compute a cosine by hand, and say when it is undefined;
- say why a raw-count cosine and a PPMI cosine can disagree;
- state what context distribution smoothing does to the denominator, and in which direction;
- expand $\mathbf{u}^\top G \mathbf{v}$ into its individual terms;
- build a G that reverses a given ranking, and verify it;
- write an outer product, give its rank, and say why;
- say what a symmetric G cannot do, and why that matters for relations.

Questions are welcome in class or by email before the quiz.